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Flow States in Math: The Relationships with Attitudes Towards Math and Engagement in the Classroom

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The purpose of this study is to analyze the relationships between the flow states and undergraduate attitudes' towards math and engagement in a mathematics course in an education faculty. The results showed that the more positive attitudes the students had towards mathematics courses, the more flow they experienced and the less anxiety they had in mathematics courses. In addition, the flow and anxiety levels of the students varied according to their major fields of study. The groups with the lowest levels of anxiety and the highest levels of flow state were students who were studying mathematics education and science education, whereas the students studying computer technology education had the highest levels of anxiety and the lowest levels of flow state. Increasing emotional engagement and positive attitudes of students in mathematics courses can help them stay longer in flow state.

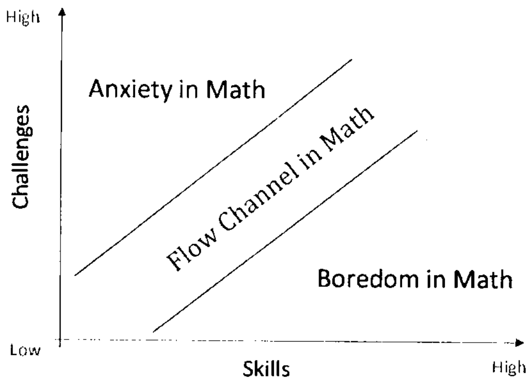
Keywords: Mathematics anxiety, attitudes toward mathematics, class engagement, flow states in math.

Introduction

People face many different problems in their daily lives. In this context, mathematics is an important scientific discipline that helps people deal with these problems. As in the rest of the world, many of the students in Turkey are anxious about learning mathematics and avoid learning mathematics as a coping strategy (Alkan, 2011; Baloğlu, 1999; Domino, 2009). As a result, these facts display themselves in a negative way. In fact, Turkish students' math performance are ranked far behind students in many other countries in international exams such as ***Trends in International Mathematics and Science Study*** (TIMSS) and ***Programme for International Student Assessment*** (PISA) (Yalçın & Tavşacıl, 2014). Moreover, problems experienced in the mathematics learning process such as negative attitude towards mathematics, math anxiety, memorizing, not being able to associate mathematics with daily life, show similar patterns in many countries (Domino, 2009; Kara & Özkan, 2016; Relich, Way & Martin 1994). Therefore, research is called for to reveals the factors enabling students to engage in mathematics courses with pleasure. In order to overcome the difficulties of teaching mathematics, flow state in mathematics courses may be a factor worthy of investigation.

"Flow" is described as a mental state where one is involved in and focused on an activity so deeply that there is a state of complete immersion in an activity and time seems to fly away (Csikszentmihalyi, 1990). In any flow experience, one uses his or her full skills and capacities when they encounter a challenge. When there is an appropriate balance between the skills and capacities and the challenge, a state of flow occurs along with a complete engagement. However, it leads to anxiety when the challenge is too high for the person's skills and it leads to boredom when the challenge is too low (Csikszentmihalyi, 1975, 1990; Csikszentmihalyi & LeFevere, 1989; Csikszentmihalyi & Rathunde, 1993; Eryılmaz & Mammadov, 2016). A review of the literature on mathematics education provides us with studies that indirectly emphasize the importance of flow experiences in mathematics teaching (Ainley, Pratt, & Hansen, 2006; English, 2006; Henningsen & Stein, 1997; Watson & De Geest, 2005; Hodge, Zhao, Visnovska, & Cobb, 2007). A flow experience has three major states: anxiety, flow and boredom (Csikszentmihalyi, 1975, 1990; Csikszentmihalyi & LeFevere, 1989; Csikszentmihalyi & Rathunde, 1993; Eryılmaz & Mammadov, 2016). There is considerable research on anxiety of mathematics (Baloğlu, 1999; Quilter & Harper, 1988; Satake & Amato, 1995; Zakaria & Nordin, 2008). However, there is yet no research examining all the three states of flow experiences in relation to flow in mathematics lessons. Conducting studies about these three states in the context of mathematics teaching may reveal more clearly the transition process between states and flow experiences as noted in Figure 1 (Csikszentmihalyi, 1990).

Figure 1: The process between states and flow experiences for math teaching



Note. **This figure adapted from Csikszentmihalyi, (1990).**

A significant factor related to a flow experience in mathematics course is engagement. The active participation of individuals in the learning-teaching process and in the activities carried out in this process is defined as student engagement in educational terms (Christenson, Reschly & Wylie, 2012). Fredricks, Blumenfeld and Paris (2004) described engagement as a multi-dimensional structure having three aspects: cognitive, behavioral, and affective. Cognitive engagement is recognition of the value of learning with a great desire beyond the minimum requirement. Affective engagement refers to student attitudes towards learning activities and involves the reactions students have towards the school, their teachers and their peers. Behavioral engagement involves the idea of active participation in academic and social activities and is considered to be very important for academic achievement (Attard, 2014; Skinner, Kindermann & Furrer, 2009).

According to Attard (2014), student engagement in mathematics courses is critical for students to develop a sense of appreciation and understanding of the value of mathematics teaching throughout their compulsory education life. Research on the relationship between mathematics courses and engagement reveals that students can learn math and be successful in mathematics if they have effective, cognitive and behavioral participation in mathematics (Ainley, Pratt, & Hansen, 2006; Boaler, 2009; Leon, Medina-Garrido, & Núñez, 2017; Lobato, Clarke, & Ellis, 2005; Marks, 2000; Steinberg, Empson, & Carpenter, 2004; Olivier, Archambault, De Clercq, & Galand, 2019; Ozkal, 2019: Sullivan et al., 2005). However, there appears to be no research that examines the relationship between flow experiences and engagement in mathematics courses.

Another concept closely related to achievement in mathematics and mathematics teaching is student attitude towards mathematics. Attitude towards mathematics is defined as the tendency of a person to exhibit positive or negative emotions, thoughts and behavior towards mathematics and subjects in mathematics (Tarım & Artut, 2016). Research showed that students' mathematical achievements increased due to their positive attitudes towards mathematics (Mazana, Suero Montero & Olifage, 2019; McGraw, Lubienski & Strutchens, 2006; Peker & Mirasyedioğlu, 2003; Peteros, Columna, Etcuban & Almerino, 2019; Recber, Isiksal & Koc, 2017; Uysal Koğ & Başer, 2012; Yenilmez, 2007). On the other hand, there seems to be no research examining the relationship between attitudes toward mathematics courses and flow experience in mathematics courses.

Both global and nationwide research found that there were a number of problems in teaching mathematics. For example, according to NCTM (2014), difficulties in learning mathematics include motivation prior to schooling, previous teaching errors, difficulties in mentally remembering mathematical concepts, underdeveloped sense of numbers, and slow or incorrect recall of basic arithmetic operations. Studies in Turkey also showed that there were problems in mathematics teaching. Durmuş (2004), for instance, made interviews with students and identified difficulties in secondary school mathematics courses as difficulties in mathematical concepts and lack of motivation. According to Tatar and Dikici (2008), the causes of difficulties in learning mathematics are deficiencies in the teaching of mathematics, the difference between the development of students' abstract thinking skills and the increase in abstractity, misconceptions in interpreting verbal expressions and problems in student readiness. Flow experiences and engagement are closely related (Shernoff, Csikszentmihalyi, Schneider & Shernoff, 2003). Investigating the relationships between flow in mathematics course and engagement and attitudes towards mathematics course in order to increase students' flow experiences could contribute to solving the problems regarding the teaching of mathematics.

Methodology

Research design

The purpose of this study is to examine the relationships between education faculty undergraduate students' flow states and their attitudes towards and engagement in a mathematics course. The study was a cross-sectional study that examined the relationships between variables by multiple regression analysis. In addition, this study examined whether the flow states of the students studying mathematics education, science education, computer technology education and elementary school education varied according to their major fields of study using one-way analysis of variance.

In addition, one-way analysis of variance was conducted to analyze whether the flow states of the students studying mathematics education, science education, computer technology education and elementary school education varied according to their major fields of study.

Research sample

In the process of sample selection, convenience sampling from purposive sampling methods was used. Convenience sampling is a method that is more frequently used by researchers because of its ease of implementation (Yıldırım and Şimşek, 2006). This study was conducted with volunteer undergraduate students studying at a state university in Turkey. The sample consisted of 227 students. These stduents were studying mathematics education (60 students, 26.4%), science education (65 students, 28.6%), computer technology education (41 students, 18.1%) and elementary school education (61 students, 26.9%). Among the participants 172 (75.8%) were female and 55 (24.2%) were male. The participants were 18-19 years old. All participants were first year students who were taking a mathematics courses at the university at the time of data collection.

Measurement tools

The Classroom Engagement Scale, the Flow State Scale in Mathematics Courses and the Mathematics Course Attitude Scale were used in the study. The psychometric properties of these scales are presented below.

The classroom engagement scale

The Classroom Engagement Scale was developed by Eryılmaz (2014) for university students. It has been developed to measure the class engagement of university students. 15 items were in the survey. The scale has three factors: behavioral engagement (e.g. asking questions, responding to questions asked, etc.), cognitive engagement (e.g. constructing the subject in the mind, relating the subject with the previous ones, etc.) and emotional engagement (e.g. curiosity and interest, etc.). There are five items for each factor. The Cronbach's Alpha reliability coefficients of the factors vary between 0.81 and 0.90. The reliability of the total score of the scale is 0.94.

The flow state scale in mathematic course. The Flow State Scale in Mathematic Lesson was developed by Eryılmaz and Mammadov (2016). It was developed to measure the flow status of high school students in mathematics courses. The scale has three factors: flow, anxiety and boredom. 12 items are in the survey. There are five items for flow, 4 items for boredom and 3 items for anxiety. The reliability values of the factor of the scale ranges from 0.83 to 0.91. In this study group, the reliability of the scale was computed. According to results, the Cronbach Alpha value for flow was 0.89; for anxiety was 0.80, and for boredom was 0.81.

The mathematics course attitude scale. The Mathematics Course Attitude Scale was developed by Aşkar (1986). This Likert type scale is used to measure students' attitudes towards mathematics lesson. The scale has 20 items consisting of 10 positive and 10 negative items. Mathematics attitude scale scored with a scale of 1-5. Students were asked to indicate their degree of participation in each item. In the Likert type scale, since the scale score is the sum of the response points shown to the items, each attitude item in the scale is scored. The documented Cronbach alpha value of the scale is 0.83. For this study, the reliability of the scale was computed to be 0.95.

Data Collection

The measurement tools prepared in printed form were applied to the undergraduate students who voluntarily participated in the study. They were asked to complete all the scales without any time limit. Participants completed the scales in any order. Each participant answered all measurement tools.

Data analysis

Descriptive analysis was conducted to describe the flow states, engagement and attitude to mathematics of the undergraduate students studying mathematics education, science education, computer technology education and elementary school education varied according to their major fields of study. Multiple regression analysis was used to determine the extent to which the classroom engagement factors accounted and attitudes towards mathematics course can explain the participants' flow states. In addition, one-way analysis of variance was conducted to analyze whether the flow states of the students studying mathematics education, science education, computer technology education and elementary school education varied according to their major fields of study.

Findings

Table 1: Descriptive Statistics

Variables		Undergraduate Student	*N*	$\overline{X}$	*SD*
Flow States	Flow	Science	65	16.04	2.85
		Mathematics	60	16.06	1.99
		Class	61	14.11	3.14
		Technology	41	13.06	2.94
	Boredom	Science	65	8.05	1.83
		Mathematics	60	7.25	2.09
		Class	61	7.66	2.18
		Technology	41	6.99	2.14
	Anxiety	Science	65	7.25	2.00
		Mathematics	60	6.71	1.85
		Class	61	7.23	2.31
		Technology	41	8.72	2.82
Engagement in Mathematics	Behavioural Engagement	Science	65	13.91	2.51
		Mathematics	60	13.32	2.59
		Class	61	13.84	2.83
		Technology	41	13.56	2.82
	Emotional Engagement	Science	65	14.74	2.71
		Mathematics	60	15.14	2.43
		Class	61	13.71	2.90
		Technology	41	12.84	3.23
	Cognitive Engagement	Science	65	13.64	2.71
		Mathematics	60	14.49	2.35
		Class	61	13.34	2.61
		Technology	41	12.65	2.65
Attitude to Mathematics	Attitudes	Science	65	74.93	13.44
		Mathematics	60	81.49	9.42
		Class	61	67.09	16.81
		Technology	41	69.98	15.52

Descriptive statistics

Descriptive statistics were given in Table 1 before examining the relationships between the flow status of education faculty students and their attitude and engagement in mathematic. Table 1 shows the flow states, engagement and attitude towards mathematics of the undergraduate students studying mathematics education, science education, computer technology education and elementary school education varied according to their major fields of study was analyzed.

According to Table 1, in the flow state, the technology group has the lowest mean in flow and boredom, while it is the group with the highest mean in anxiety. In engagement in mathemetics, although the mean of all groups in behavioral engagement is almost the same, it is the lowest mean technology group in cognitive and emotional engagement. Attitude to mathematics mean of class and technology groups are lower than science and mathematics groups (see Table1).

Regression analysis results

Multiple regression analysis was conducted to determine the extent to which the classroom engagement factors accounted for

flow states. In addition, flow states and attitudes towards mathematics courses were also examined. Tables 2 and 3 show the results.

Table 2: Multiple Regression Analysis Results: Engagement and Flow States

Engagement Dimension	Variables			
Flow	B	SEB	Beta	t
Behavioural Engagement	.01	.05	.01	.27
Emotional Engagement	.67	.06	.65	10.65**
Cognitive Engagement	.12	.06	.11	1.87
Boredom	B	SEB	Beta	
Behavioural Engagement	.05	.05	.06	.92
Emotional Engagement	-.11	.06	-.15	-1.72
Cognitive Engagement	.01	.06	.01	.20
Anxiety	B	SEB	Beta	
Behavioural Engagement	.07	.05	.08	1.26
Emotional Engagement	-.31	.06	-.40	-5.03**
Cognitive Engagement	-.13	.06	-.15	-1.97

Table 2 shows the relationships between the factors of flow process and classroom engagement factors. There was a significant relationship between flow state and the classroom engagement factors (R=0.73, R^2=0.54, F=88.08, p<0.01). The emotional engagement factor significantly accounted for flow (β=.65; t=10.65, p=.00). No significant relationship was found between boredom state and the engagement factors (R=0.12, R^2=0.02, F=1.26, p>0.01). There was a significant relationship between anxiety state and the engagement factors (R=0.48, R^2=0.23, F=21.81, p<0.01). Only emotional engagement significantly accounted for anxiety (β= -.40; t = -5.03, p =0.00).

Table 3: *Regression Analysis Results: Flow States and Attitudes of Mathematics*

Dimensions	Variables			
Flow	B	SEB	Beta	t
Attitudes	.13	.01	.66	13.40**
Boredom	B	SEB	Beta	
Attitudes	-.00	.00	-.01	-.14
Anxiety	B	SEB	Beta	
Attitudes	-.08	.00	-.56	-10.18**

Table 3 shows the relationships between the factors of flow process and attitudes towards mathematics course. There was a significant relationship between flow state and attitudes towards mathematics courses ($R=0.66$, $R^2=0.44$, $F=179.61$, $p<0.01$). Attitudes towards mathematics courses significantly accounted for flow state ($\beta= 0.66$; $t=13.40$, $p =0.00$). No significant relationship was found between boredom state and attitudes towards mathematics course. There was a significant relationship between anxiety state and attitudes towards mathematics course ($R=0.56$, $R^2=0.31$, $F=103.79$, $p<0.01$). Attitudes towards mathematics significantly accounted for anxiety ($\beta= -0.56$; $t = -10.18$, $p =0.00$).

Variance analysis results

In this study, whether the flow states of the students varied according to their major fields of study, mathematics education, science education, computer technology education, and elementary school education, was also analyzed by one-way analysis of variance. The results indicated that, experiencing flow states in mathematics course varied according to the major fields of study ($F=223.3, 14.87$; $p<0.01$). According to the Tukey test results, the participants studying science education ($\overline{X}=16.04$; $p<0.01$) and mathematics education.

($\overline{X}$ =16.06; $p<$ 0.01) had higher average scores of flow than the participants studying elementary school education ($\overline{X}$ =14.11, $p<0.01$) and computer technology education. There was no variation among the major fields of study in terms of boredom state in mathematics course. On the other hand, experiencing anxiety in mathematics course varied according to the major fields of study (F= 223.3, 6.92; $p<0.01$). According to the Tukey test results, the participants studying computer technology education had higher average scores of anxiety ($\overline{X}$ = 8.72) than the participants studying science education ($\overline{X}$ =7.25, $p<0.01$), elementary school education ($\overline{X}$ =7.23, $p<0.01$) and mathematics education ($\overline{X}$ =6.71, $p<0.01$).

Figure 2: *Scores of flow*

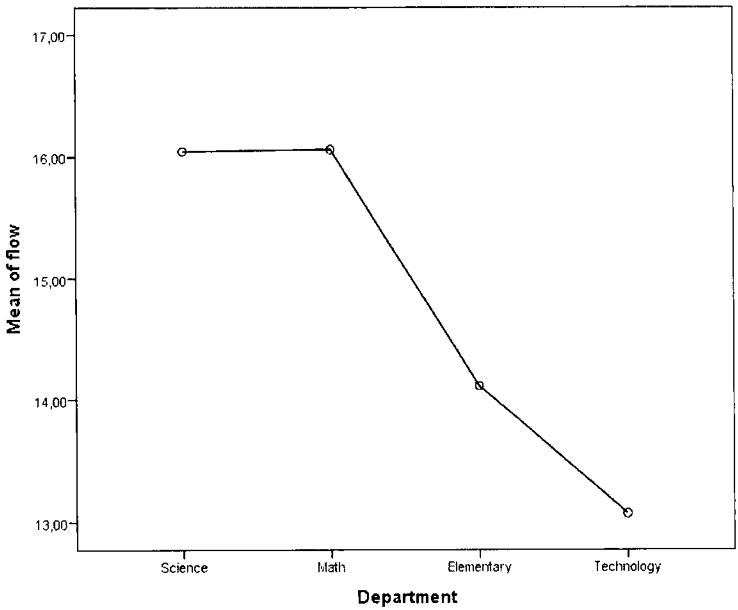
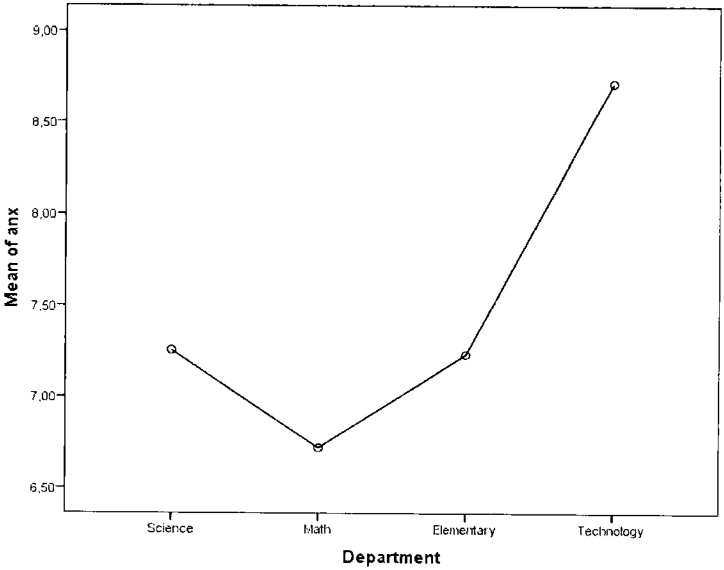


Figure 3:*Scores of anxiety*



Discussion and Conclusion

Research suggests that the tasks assigned to students in relation to mathematics affect students' learning of mathematics, and understanding of the nature of mathematics (Ainley, Pratt, & Hansen, 2006; Hodge, Zhao, Visnovska, & Cobb, 2007; Remillard, & Heck, 2014; Torbeyns, Schneider, Xin, & Siegler, 2015). Researchers in particular argue that challenging tasks affect and improve students' mathematical thinking (English, 2006; Henningsen & Stein, 1997; Lo, Hew, & Chen, 2017; Watson & De Geest, 2005). In other words, researchers indirectly point to the importance of flow experience. However, there is still limited research measuring flow experience in mathematics course and exploring its relationships with the related variables (Eryılmaz & Mammadov, 2016). We hope this study could be an important contribution as it provides a description for these issues in the literature.

The results of the study showed that, in parallel with the increase in the level of emotional engagement and positive attitudes of the undergraduate students, there were more flow experiences and less anxiety experiences in mathematics courses. Research highlighted the importance of emotional engagement in mathematics teaching. Marks (2000), for example, stated that students found the school more rewarding when their emotional engagement in mathematics courses was higher. In fact, in a study by Sullivan et al. (2005), some of the students turned to options related to mathematics for their career choices in parallel to the increase in emotional engagement in their mathematics courses. Similarly, Boaler (2009) argued that negative direction of emotional engagement was the reason why many students hated mathematics. Our results confirm these results as our study showed the presence of a relationship between emotional engagement and flow and anxiety in mathematics courses. This study also showed that emotional engagement could be an important tool for reducing students' anxiety and increasing flow states in mathematics courses.

Research often showed the need for teachers to promote their students' behavioral engagement in learning mathematics such as guiding students through methods in mathematical operations (Ainley, Pratt, & Hansen, 2006), asking students questions and giving answers to them (Steinberg, Empson, & Carpenter, 2004), listening to students (Lobato, Clarke, & Ellis, 2005) and increasing behavioral engagement by means of group tasks (O'Conner & Michaels, 1996). Unlike these results, the results from our study revealed the importance of emotional engagement in mathematics learning. In this sense, our results provide important implications. For example, in mathematics teaching, it may be functional for teachers to create teaching processes that enhance students' emotional engagement in the classroom environment.

The results from this study are also important in terms of revealing the relationship between flow and engagement. Shernoff et al. (2003) stated that flow experience itself contains engagement and, in this respect, engagement included properties such as concentration, enjoyment and interest. In other words, they only emphasized emotional aspect of engagement. On the other hand, there are three major aspects of engagement according to the literature: emotional, cognitive and behavioral (Christenson, Reschly & Wylie, 2012; Eryılmaz, 2014, Fredricks, Blumenfeld & Paris, 2004, Skinner, Kindermann & Furrer, 2009). Showing the highly significant relationship between emotional engagement and flow experience, our results not only confirmed the findings of Shernoff et al. (2003) but also revealed the need for examining engagement more elaborately (i.e. by involving the relationships between behavioral and cognitive aspects). The reason for this is that, in this study, flow experience in mathematics course was only related with emotional engagement not with cognitive or behavioral engagement. In this sense, it would be fair to state that this study contributed to the flow theory.

According to the results of this study, emotional engagement comes before behavioral and cognitive engagement in the flow in mathematics lessons. According to Piaget (1970), one of the biggest theorists explaining cognitive development, emotions are the fuel of behavior (cited in Wadsworth, 1996). The results empirically support applying Piaget's theoretical explanations.

The results of this study also provide implications for the functions of emotions. According to Fredrickson (2001), positive emotions have three important functions. First, positive emotions help individuals become better problem solvers by expanding their perspectives. Second, positive emotions build individuals' capacities. Third, positive emotions heal previous negative experiences. In terms of engagement, the engagement indicator showing positive emotions is emotional engagement. It is possible that functions of positive emotions emerged when our participants were experiencing emotional engagement. This may have let the participants to move away from their anxiety and experience flow in mathematics courses.

According to the results of this study, the more positive attitudes the students had towards mathematics course, the more flow they experienced and the less anxiety they had in mathematics courses. This result is consistent with the existing literature. For example, according to Lewis (2013), students' emotions and their negative attitudes towards mathematics are highly correlated in mathematics learning process. Since the attitude of an individual depends on experiences, attitude is a changeable concept. Evidence showed that using different instructional strategies such as discovery learning and problem-solving learning and teacher attitudes positively influenced student attitudes toward mathematics (Akinsola & Olowojaiye, 2008; Bal İncekabak & Ersoy, 2016; Domino, 2009; Kara & Özkan, 2016; Relich, Way & Martin 1994). However, no previous research examined attitudes towards mathematics courses with respect to flow experiences. In this sense, it would be fair to state that this study contributed to the literature.

In our study, the flow and anxiety states of the students varied according to their major fields of study. The groups with the lowest levels of anxiety and the highest levels of flow state were students who were studying mathematics education and science education, whereas the students studying computer technology education had the highest levels of anxiety and the lowest levels of flow state. In fact, the theory of flow seems to suggest an answer for this result. The theory of flow holds that individuals can experience more flow and less anxiety through a balance between the challenges of an activity and the skills required to meet them (Csikszentmihalyi, 1975, 1990; Csikszentmihalyi & LeFevere, 1989; Csikszentmihalyi & Rathunde, 1993; Eryılmaz & Mammadov, 2016). In Turkey, subject teachers are selected through a central examination system. In these exams, students are required to answer questions about their area of study. Students studying mathematics education and science education receive higher scores in science and mathematics than students studying elementary school education or computer technology education (Karakaya, 2011). What is more, students of technology education tend to come from vocational high schools, where the curriculum often places less emphasis on mathematics (Usun, 2003). Therefore, it can be said that the students studying mathematics education and science education prepared for the central exams developed the necessary skills against the difficulties of mathematics. Thus, these students can be said to be less anxious and have more flow in mathematics classes than the students studying elementary school education and computer technology education.

In conclusion, this study empirically examined the relationship between flow experience and attitude towards and engagement in mathematics courses, and the flow states of undergraduate students in different academic fields. The results of the study revealed interesting findings. The flow and anxiety levels of the students varied according to their major fields of study. The groups with the lowest levels of anxiety and the highest levels of flow state were students who were studying mathematics education and science education, whereas the students studying computer technology education had the highest levels of anxiety and the lowest levels of flow state. The study is different from the previous research as it was conducted with undergraduate students. A number of recommendations can be made based on the results of the study. First of all, striking a balance between the challenges of an activity and the skills required to meet them is also important for flow experience, and these points should be taken into account in mathematics teaching. In addition, it is vital to increase the emotional engagement of students so that they can experience flow in mathematics courses. Ensuring that students hold positive attitudes to mathematics courses can help them stay in flow state longer. There are many factors that affect students' mathematics learning and their attainment in mathematics. Some of these factors are related to schools and classes (e.g. classroom engagement) while others are considered as out-of-school factors (Attard, 2012, 2013; Boaler, 2009;

Bragg, 2012; Hopkins, 2008; Ricks, 2009). Future research can examine the relationship between flow experience and out-of-school factors in mathematics courses.

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